

**Exam 2 (Sections 5.7- 6.2)**

**Directions:** Please show all your work neatly and clearly. You will not receive full credit unless you show all work. Unless otherwise indicated, each problem is worth 6 points.

1. Determine the following. If the integral is definite, please leave your answer in simplified exact form.

a. 
$$\int_0^{\pi/2} \frac{\cos x}{4 + \sin^2 x} dx$$

b. 
$$\int \frac{2x - 3}{\sqrt{1 - x^2}} dx$$

c. 
$$\int \frac{1}{(x - 2)\sqrt{x^2 - 4x}} dx$$

d.  $\int \frac{\operatorname{sech}(1/x) \tanh(1/x)}{x^2} dx$

e.  $\int_0^4 \frac{1}{25-x^2} dx$

f.  $\int \frac{dx}{x^2 - 4x + 7}$

g.  $\int \frac{x}{\sqrt{x^4 - 1}} dx$

2. Determine the derivative:

a.  $f(x) = \tanh(2x^3 + 1)$

b.  $g(t) = \arctan(\sinh t)$

c.  $y = \cosh^{-1}(3x)$

3. Determine whether the function  $y = x^3$  is a solution of the differential equation  $x^2y' + 3y = 6x^3$

4. Find the principal  $P$  that must be invested at the rate of 7%, compounded continuously, so that \$500,000 is available after 20 years (for Jay to retire).

5. **(10 pts)** In some chemical reactions, the rate at which the amount of a substance changes with time is proportional to the amount present. For the change of  $\delta$ -gluconolactone into gluconic acid, for example,

$$\frac{dy}{dt} = -0.6y$$

when  $t$  is measured in hours.

- a. Solve the differential equation to determine  $y$  as a function of  $t$ .
- b. If there are 100 grams of  $\delta$ -gluconolactone present when  $t = 0$ , how many grams will be left after the first hour?
6. **(9 points)** Find the particular solution of the differential equation:

$$y \frac{dy}{dx} = e^x \text{ with initial conditions: } y(0) = 4$$

7. **(9 points)** Solve the differential equation  $y' = x(1 + y)$